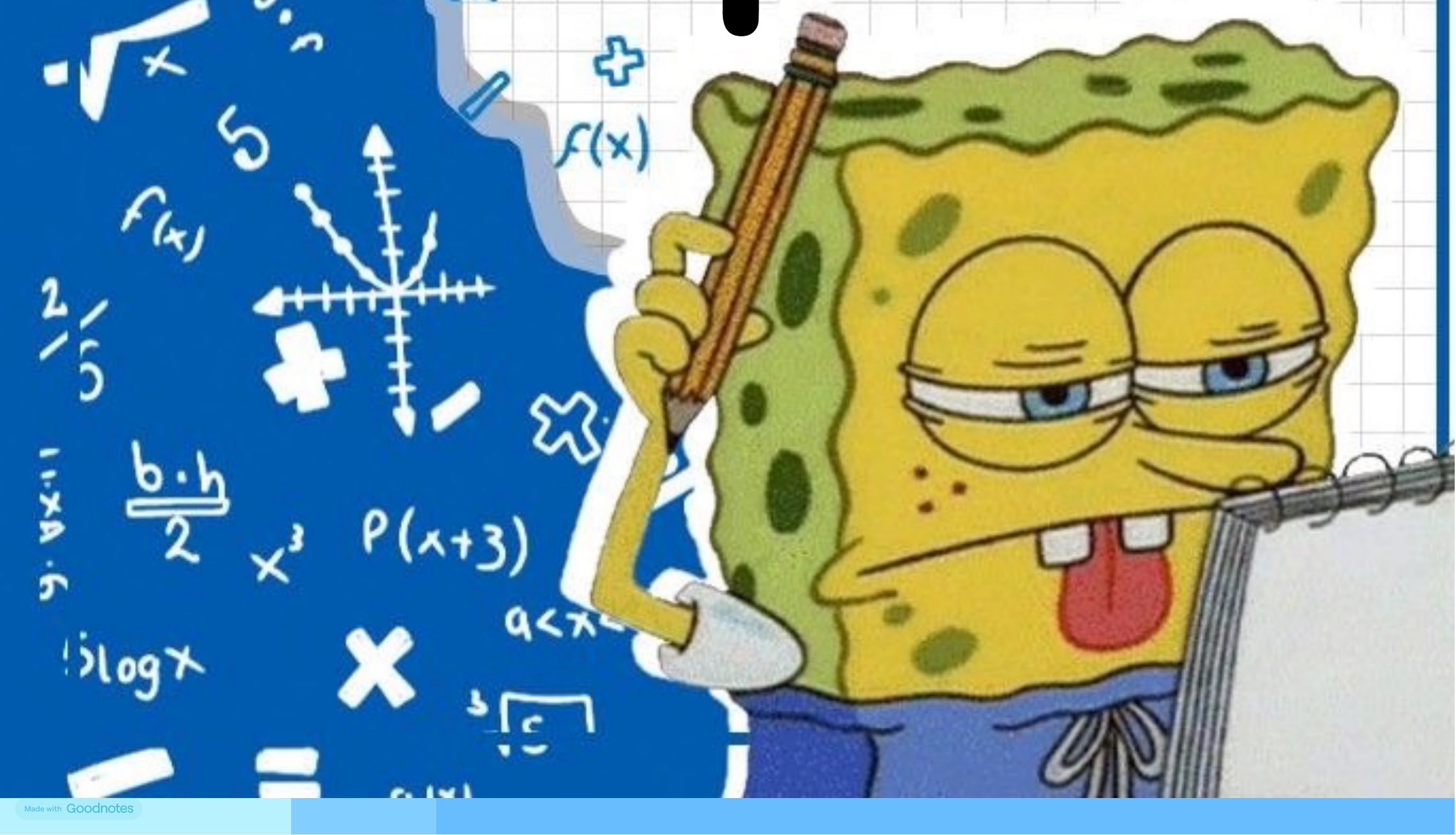
SEMANA 14

Matemáticas

4



I. SEGUNDO TEOREMA DE TRASLACIÓN

En los ejercicios 1 - 16, determinar L { f (t)}

1) $f(t)=(t-5)^4 e^{t-5}u(t-5)$

$=\mathcal{L}\{(t-5)^4 e^{t-5}u(t-5)\}$ → $e^{-5s}\mathcal{L}\{e^t t^4\}$ aplicando el 1° T. de Traslación

$f(t-5)$ $a=5$

$=e^{-5s}\mathcal{L}\{e^{t-5}(t-5)^4\}$ $= e^{-5s}\left[\mathcal{L}\{t^4\}|_{s=s-1}\right]$

$f(t-5)=(t-5)^4$ $= e^{-5s}\left[\frac{4!}{s^5}|_{s=s-1}\right]$

$f(t)=t^4$ $= e^{-5s}\left[\frac{24}{s^5}|_{s-1}\right] = e^{-5s}\left(\frac{24}{(s-1)^5}\right)$

$F(s)=\frac{24e^{-5s}}{(s-1)^5}$

2) $f(t)=te^{t-4}u(t-4)$ $a=4$

$=\mathcal{L}\{t-4+4\,e^{t-4}u(t-4)\}$ $e^{-4s}\mathcal{L}\{t+4\}|_{s=s-1}$

$=e^{-4s}\mathcal{L}\{(t-4)+4\cdot e^{t-4}\}$ $e^{-4s}\left[\frac{1}{s^2}+\frac{4}{s}\right]$

$=e^{-4s}\mathcal{L}\{(t+4)e^t\}$ aplicando 1° T.T $e^{-4s}\left(\frac{1+4s}{s^2}\right)$ → $e^{-4s}\left[\frac{-3+4s}{(s-1)^2}\right]$ = $F(s)=\frac{-3e^{-4s}+4se^{-4s}}{(s-1)^2}$

$s=s-1$

$e^{-4s}\mathcal{L}\{t+4\}|_{s=s-1}$

3) $f(t)=(t-1)^2 e^{1-t}u(t-1)$ → $\mathcal{L}\{(t-1)^2 e^{1-t}u(t-1)\}$

$R/F(s)=\frac{2e^{-s}}{(s+1)^3}$

$a=1$

→ $e^{-s}\mathcal{L}\{(t-1)^2 e^{1-t}\}$

→ $e^{-s}\mathcal{L}\{(t-1)^2 e^{-(t-1)}\}$ → $F(s)=\frac{2e^{-s}}{(s+1)^3}$

→ $e^{-s}\mathcal{L}\{t^2e^{-t}\}$

$t=t-1$ aplicando 1 teorema de translacion

$s=s+1$ $\mathcal{L}\{t^2\}$

$=\frac{2}{s^3}\Big|_{s+1}=\frac{2}{(s+1)^3}$

4) $f(t)=e^{3(t-2)}sen(t-2)u(t-2)$

sea $a=2$ $s=s-3$

$t=t-2$

$F(s)=\frac{e^{-2s}}{(s-3)^2+1}$

aplicando 1 teorema de translacion

$e^{-2s}\mathcal{L}\{sen(t)e^{3t}\}$ → $e^{-2s}\mathcal{L}\{sen(t)\}|_{s-3}$

→ $e^{-2s}\left[\frac{1}{s^2+1}|_{s-3}\right]$

5) $f(t)=(4t+2)u(t-4)$

$R/F(s)=\frac{2e^{-4s}}{s^2}(2+9s)$

$\mathcal{L}\{(4t+2)u(t-4)\}$ $a=4$ sea $t=t-4$

$e^{4s}\mathcal{L}\{4((t-4)+4)+2\}$

$=e^{-4s}\mathcal{L}\{4t+18\}$ $e^{-4s}\left[\frac{4}{s^2}+\frac{18}{s}\right]=e^{-4s}\left(\frac{4+18s}{s^2}\right)$

$=\mathcal{L}\{4t+18\}$ $F(s)=\frac{2e^{-4s}}{s^2}(2+9s)$

6) $f(t)=e^{6-t}u(t-6)$ $a=6$

$R/F(s)=\frac{e^{-6s}}{s+1}$

$\mathcal{L}\{1\cdot e^{-(t-6)}u(t-6)\}$

$e^{-6}\mathcal{L}\{e^{-t}\}$

$=\frac{1}{s+1}\cdot e^{-6s}$ → $F(s)=\frac{e^{-6s}}{s+1}$

7) $f(t)=u(t-1)+2u(t-3)-6u(t-4)$

$=\mathcal{L}\{1u(t-1)+2u(t-3)-6u(t-4)\}$

$a=1$ $a=3$ $a=4$

$=e^{-s}\mathcal{L}\{1\}+2e^{-3s}\mathcal{L}\{1\}-6e^{-4s}\mathcal{L}\{1\}$

$F(s)=\frac{e^{-s}}{s}+\frac{2e^{-3s}}{s}-\frac{6e^{-4s}}{s}$

Made with Goodnotes

8) $f(t)=(t-3)u(t-2)-(t-2)u(t-3)$

$R/F(s)=e^{-3s}\left(\frac{e^{s}(1-s)-(1+s)}{s^{2}}\right)$

reajustando la expresion para una mejor comodidad de desarrollo

$t-3=[(t-2)-1]$

$t-2=[(t-3)+1]$

$f(t)=[(t-2)-1\cdot u(t-2)]-[(t-3)+1\cdot u(t-3)]$

$f(t)=(t-2)u(t-2)-u(t-2)-(t-3)u(t-3)+u(t-3)$

$=\mathcal{L}\{(t-2)u(t-2)\}-\mathcal{L}\{u(t-2)\}-\mathcal{L}\{(t-3)u(t-3)\}+\mathcal{L}\{u(t-3)\}$

$=e^{-2s}\mathcal{L}\{t\}-e^{-2s}\mathcal{L}\{1\}-e^{-3s}\mathcal{L}\{t\}+e^{-3s}\mathcal{L}\{1\}$

$=\frac{e^{-2s}}{s^{2}}-\frac{e^{-2s}}{s}-\frac{e^{-3s}}{s^{2}}+\frac{e^{-3}}{s}\rightarrow=$

$e^{-2s}\frac{(1-s)}{s^{2}}-e^{-3s}\left(\frac{1+s}{s^{2}}\right)=F(s)=e^{-3s}\left[\frac{e^{s}(1-s)-(1+s)}{s^{2}}\right]$

9) $f(t)=t^{2}u(t-\pi)$

$R/F(s)=\frac{e^{-\pi s}}{s^{3}}\left(2+2\pi s+\pi^{2}s^{2}\right)$

$\mathcal{L}\{t^{2}u(t-\pi)\}\quad a=\pi\quad t=t-\pi$

$t^{2}=[(t-\pi)+\pi]^{2}$

$t^{2}=(t+\pi)^{2}$

$=t^{2}+2\pi t+\pi^{2}$

$=e^{-\pi s}\mathcal{L}\{t^{2}+2\pi t\pi^{2}\}$

$=\left(\frac{2}{s^{3}}+\frac{2\pi}{s^{2}}+\frac{\pi^{2}}{s}\right)e^{-\pi s}$

$F(s)=e^{-\pi s}\left(\frac{2+2s\pi+(\pi s)^{2}}{s^{3}}\right)$

10) $f(t)=sen(2t)u(t-\pi)$

$\mathcal{L}\{sen(2t)u(t-\pi)\}\quad a=\pi=e^{-\pi s}\mathcal{L}\{sen(2t)$

Pero $t=t+\pi-\pi$

$t=(t-\pi)+\pi$

t

$=e^{-\pi s}\mathcal{L}\{sen(2t+2\pi)\}$

$=e^{-\pi}\mathcal{L}\{sen(2t)\}=F(s)=e^{-\pi s}\left(\frac{2}{s^{2}+4}\right)$

$sen(2t)\cos(2\pi)+\cos(2t)sen(2\pi)$ (1, 0)

$sen(2t)$

12) $f(t)=\cos(2t)u(t-\pi)$

$R/F(s)=\frac{se^{-\pi s}}{s^{2}+4}$

$e^{-\pi s}\mathcal{L}\{\cos(2t+2\pi)\}=e^{-\pi s}\mathcal{L}\{\cos(2t)\}$

$t=t+\pi$

$\cos(2t+2\pi)$

$=\cos(2t)\cos(2\pi)-sen(2t)sen(2\pi)$

$F(s)=\frac{e^{-\pi s}s}{s^{2}+4}$

16) $f(t)=\cos^{2}(t)\,u\left(t-\frac{\pi}{2}\right)$

$R/F(s)=\frac{2e^{-\frac{\pi}{2}s}}{s^{2}+4}$

$\mathcal{L}\{\cos^{2}(t)u(t-\pi/2)\}$

$e^{-\frac{\pi}{2}s}\mathcal{L}\{\cos^{2}(t+\pi/2)\}$

$e^{-\frac{\pi}{2}s}\mathcal{L}\left\{\frac{1+\cos(2t+2\pi)}{2}\right\}\rightarrow e^{-\frac{\pi}{2}s}\frac{1}{2}\mathcal{L}\{1-\cos(2t)\}$

$\frac{e^{-\frac{\pi}{2}s}}{2}\left(\frac{1}{s}-\frac{s}{s^{2}+4}\right)$

$\frac{e^{-\frac{\pi}{2}s}}{2}\left(\frac{s^{2}+4-s^{2}}{s(s^{2}+4)}\right)$

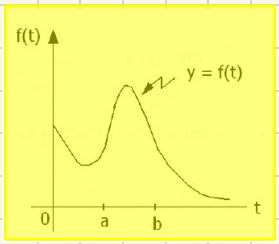
$\cos(2t+2\pi)$

$=\cos(2t)\cos(2\pi)+sen(2t)sen(2\pi)$ (0)

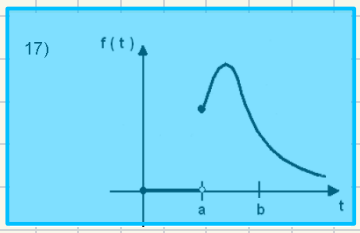
$F(s)=\frac{2e^{-\frac{\pi}{2}s}}{s(s^{2}+4)}$

Made with Goodnotes

Para los ejercicios 17 – 22, haga corresponder cada gráfica con una de las funciones de (a) a (f) .



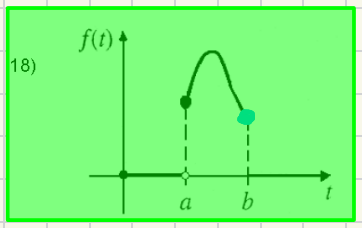
17)



$$f(t) = \begin{cases} 0 \; , & 0 \le t < a \\ f(t) & t \ge a \end{cases}$$

$$f(t) = f(t)\,u(t-a)$$

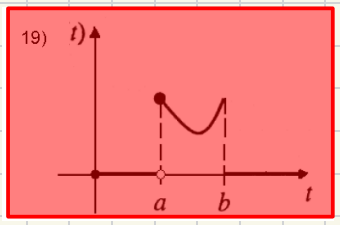
18)



$$f(t) = \begin{cases} 0 \; ; & 0 \le t < a \\ f(t) & a \le t \le b \\ 0 \; ; & t \ge b \end{cases}$$

$$f(t) = f(t)[u(t-a) - u(t-b)]$$
$$= f(t)\,u(t-a) - f(t)\,u(t-b)$$

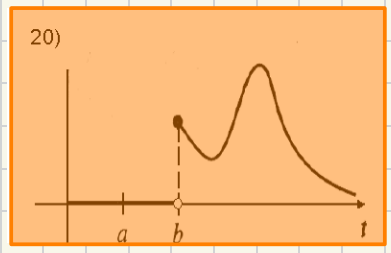
19)



$$f(t) = \begin{cases} 0 \; ; & 0 \le t < a \\ f(t-a) \; ; & a \le t \le b \\ 0 \; ; & t > b \end{cases}$$

$$f(t) = f(t-a)\,u(t-a) - f(t-a)\,u(t-b)$$

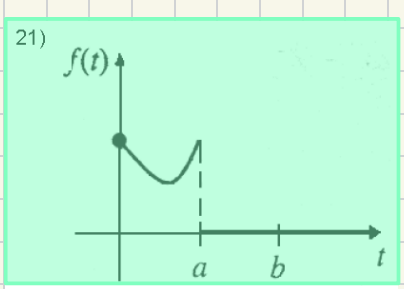
20)



$$f(t) = \begin{cases} 0 \; ; & 0 \le t \le b \\ f(t-b) & t \ge b \end{cases}$$

$$f(t) = f(t-b)\,u(t-b)$$

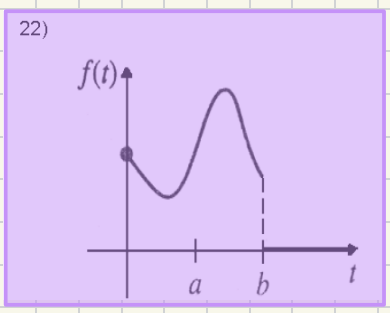
21)



$$f(t) = \begin{cases} f(t) \; ; & 0 \le t \le a \\ 0 \; ; & t \ge 0 \end{cases}$$

$$f(t) = f(t)\,u(t-0) - f(t)\,u(t-a)$$
$$= f(t) - f(t)\,u(t-a)$$

22)



$$f(t) = \begin{cases} f(t) & 0 \le t \le b \\ 0 \; , & t \ge b \end{cases}$$

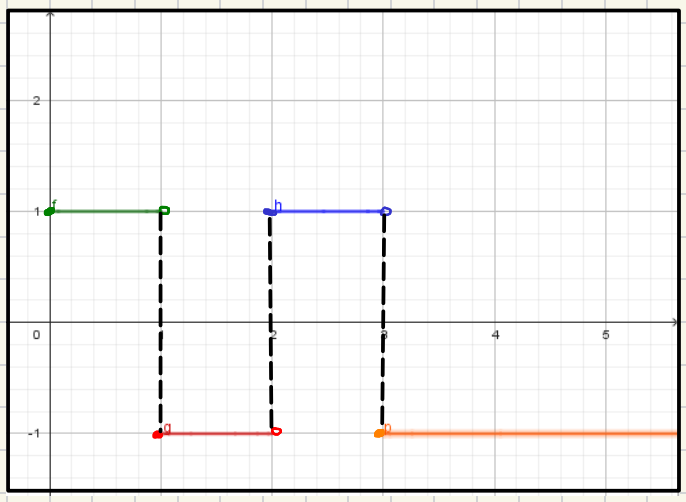
$$f(t) = f(t) - f(t)\,u(t-b)$$

En los ejercicios 23 y 24 graficar las funciones dada y escribirlas en términos de la función Escalón unitario.

23

$$f(t) = \begin{cases} 1 \; , & 0 \le t < 1 \\ -1 \; , & 1 \le t < 2 \\ 1 \; , & 2 \le t < 3 \\ -1 \; , & t \ge 3 \end{cases}$$

$$f(t) = (1)[u(t-0) - u(t-1)] - (1)[u(t-1) - u(t-2)] + 1[u(t-2) - u(t-3)] - (1)[u(t-3)]$$
$$= u(t) - u(t-1) - u(t-1) + u(t-2) + u(t-2) - u(t-3) - u(t-3)$$
$$f(t) = 1 - 2u(t-1) + 2u(t-2) - 2u(t-3)$$



$$f(t) = 1 - 2u(t-1) + 2u(t-2) - 2u(t-3)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{1 - 2u(t-1) + 2u(t-2) - 2u(t-3)\}$$
$$\underset{a=1}{2u(t-1)} \quad \underset{a=2}{2u(t-2)} \quad \underset{a=3}{2u(t-3)}$$

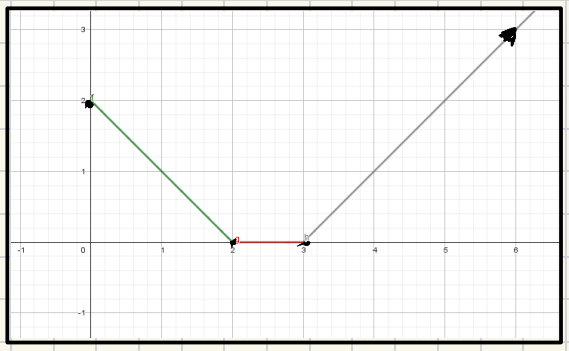
$$= \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{2e^{-2s}}{s} - \frac{2e^{-3s}}{s}$$

$$F(s) = \frac{1}{s}\left(1 - 2e^{-s} + 2e^{-2s} - 2e^{-3s}\right)$$

24)

$$f(t) = \begin{cases} 2-t \; , & 0 \le t < 2 \\ 0 \; , & 2 \le t < 3 \\ t-3 \; , & t \ge 3 \end{cases}$$

$R/\ f(t) = 2 - t + (t-2)u(t-2) + (t-3)u(t-3)$



$$f(t) = (2-t)[u(t-0) - u(t-2)] + (t-3)[u(t-3)]$$
$$= (2-t)[u(t) - u(t-2)] + (t-3)\,u(t-3)$$
$$= (2-t)(1 - u(t-2)) + (t-3)\,u(t-3)$$
$$= -(t-2)(1 - u(t-2)) + (t-3)\,u(t-3)$$
$$= -(t-2) + (t-2)u(t-2) + (t-3)\,u(t-3)$$
$$= 2 - t + (t-2)u(t-2) + (t-3)\,u(t-3)$$
$$\underset{a=2}{} \qquad \underset{a=3}{}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{(2-t) + (t-2)u(t-2) + (t-3)\,u(t-3)\}$$
$$= \mathcal{L}\{2\} - \mathcal{L}\{t\} + e^{-2s}\mathcal{L}\{t\} + e^{-3s}\mathcal{L}\{t\}$$

$$= \frac{2}{s}\frac{s}{s} - \frac{1}{s^2} + \frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2} = \frac{2s - 1 + e^{-2s} + e^{-3s}}{s^2}$$

$$F(s) = \frac{2s - 1 + e^{-2s} + e^{-3s}}{s^2}$$

Made with Goodnotes

En los ejercicios 25 y 26 escribir en términos de funciones escalón unitario, la función f cuya grafica es la siguiente.

$$f(t)=\begin{cases} t & ;\ 0\le t\le 1 \\ 2-t & ;\ 1\le t\le 2 \\ t-2 & ;\ 2\le t\le 3 \\ 4-t & ;\ 3\le t\le 4 \\ 0 & ;\ t\ge 4 \end{cases}$$

$$f(t)=(t)[u(t-0)-u(t-1)]-(t-2)[u(t-1)-u(t-2)]+(t-2)[u(t-2)-u(t-3)]-(t-4)[u(t-3)-u(t-4)]$$

$$=t\,u(t)-t\,u(t-1)-(t-2)u(t-1)+(t-2)u(t-2)+(t-2)u(t-2)-(t-2)u(t-3)-(t-4)u(t-3)+(t-4)u(t-4)$$

$$=t-(t-1+1)u(t-1)-(t-1-1)u(t-1)+2(t-2)u(t-2)-(t-2-1+1)u(t-3)-(t-4+1-1)u(t-3)+(t-4)u(t-4)$$

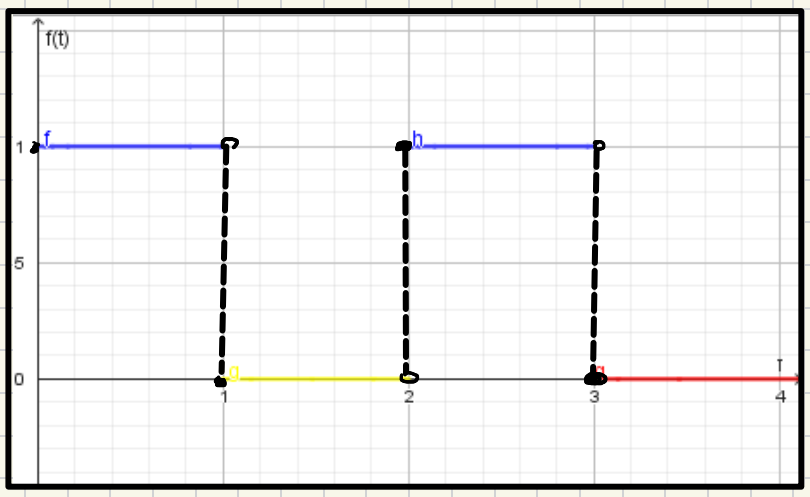
$$=t-(t-1)u(t-1)-u(t-1)-(t-1)u(t-1)+u(t-1)+2(t-2)u(t-2)-(t-3)u(t-3)-u(t-3)-(t-3)u(t-3)+u(t-3)+(t-4)u(t-4)$$

$$=t-2(t-1)u(t-1)+2(t-2)u(t-2)-2(t-3)u(t-3)+(t-4)u(t-4)$$

= APLICANDO TRANSFORMADA DE LAPLACE

$$F(s)=\frac{1}{s^2}-\frac{2e^{-s}}{s^2}+\frac{2e^{-2s}}{s^2}-\frac{2e^{-3s}}{s^2}+\frac{e^{-4s}}{s^2}$$

$$F(s)=\frac{1}{s^2}\left(1-2e^{-s}+2e^{-2s}-2e^{-3s}+e^{-4s}\right)$$



$$f(t)=\begin{cases} 1 & ;\ 0\le t<1 \\ 0 & ;\ 1\le t<2 \\ 1 & ;\ 2\le t<3 \\ 0 & ;\ t\ge 3 \end{cases}$$

$$f(t)=(1)[u(t-0)-u(t-1)]+1[u(t-2)-u(t-3)]$$

$$f(t)=u(t)-u(t-1)+u(t-2)-u(t-3)$$

$$f(t)=1-\underset{a=1}{1u(t-1)}+\underset{a=2}{u(t-2)}-\underset{a=3}{u(t-3)}$$

APLICANDO TRANSFORMADA DE LAPLACE

$$F(s)=\frac{1}{s}-\frac{e^{-s}}{s}+\frac{e^{-2s}}{s}-\frac{e^{-3s}}{s}$$

$$F(s)=\frac{1}{s}\left(1-e^{-s}+e^{-2s}-e^{-3s}\right)$$

En los problemas 27 al 31 expresar cada función en términos de la función escalón unitario. Además, hallar L { f (t)} y Graficar las funciones para mejor compresion.

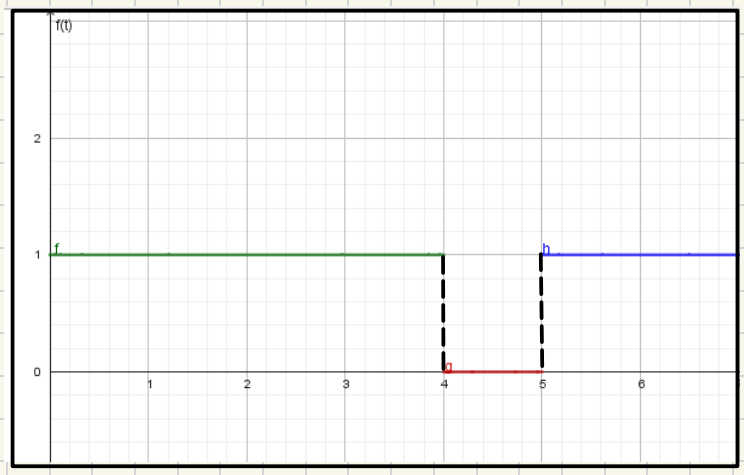
27) $$f(t)=\begin{cases} 1, & 0\le t<4 \\ 0, & 4\le t<5 \\ 1, & t\ge 5 \end{cases}$$

$$f(t)=(1)[u(t-0)-u(t-4)]+(1)[u(t-5)]$$

$$=u(t)-u(t-4)+u(t-5)$$

$$=1-\underset{a=4}{1u(t-4)}+\underset{a=5}{1u(t-5)}$$

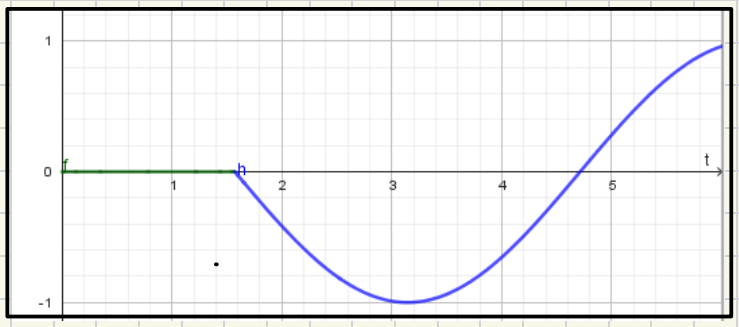
Aplicando transformada de laplace a ambos miembros.



$$F(s)=\frac{1}{s}-\frac{e^{-4s}}{s}+\frac{e^{-5s}}{s}\longrightarrow F(s)=\frac{1}{s}\left(1-e^{-4s}+e^{-5s}\right)$$

Made with Goodnotes

28) $f(t)=\begin{cases}0, & 0\le t<\dfrac{\pi}{2}\\ \cos(t), & t\le\dfrac{\pi}{2}\end{cases}$



$f(t) = \cos(t)\,u(t-\pi/2)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos(t)\,u(t-\pi/2)\}$

$= e^{-\frac{\pi}{2}s}\mathcal{L}\{\cos(t-a)\}$

$= -e^{-\frac{\pi}{2}s}\mathcal{L}\{\operatorname{sen}(t)\}$

$a=\frac{\pi}{2}$

$t = t+\frac{\pi}{2}-\frac{\pi}{2}$

$t = (t-\frac{\pi}{2})+\frac{\pi}{2}$

$t = t+\frac{\pi}{2}$

$\cos(t+\pi/2) = \cos(t)\cos(\pi/2) - \operatorname{sen}(t)\operatorname{sen}(\pi/2)$

$\cos(t+\pi/2) = -\operatorname{sen}(t)$

$$F(s) = -\frac{e^{-\frac{\pi}{2}s}}{s^2+1}$$

29) $f(t)=\begin{cases}t, & 0\le t<2\\ t^2, & t\ge 2\end{cases}$

$f(t) = t[u(t-0)-u(t-2)] + t^2[u(t-2)]$

$= t - tu(t-2) + t^2u(t-2)$

$= t - (t+2)u(t-2) + (t+2)^2u(t-2)$

$a=2$ $\qquad a=2$

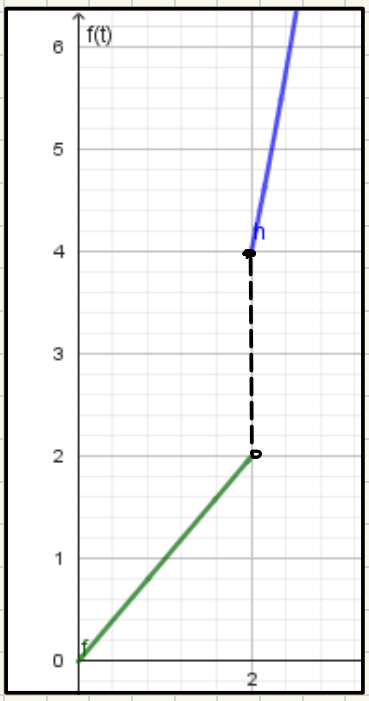
$= t - (t+2) + (t^2+4t+4)$

$= \frac{1}{s^2} - e^{-2s}\left[\frac{1}{s^2}+\frac{2}{s}\right] + e^{-2s}\left[\frac{2}{s^3}+\frac{4}{s^2}+\frac{4}{s}\right]$

$= \frac{1}{s^2} - \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s} + \frac{2e^{-2s}}{s^3} + \frac{4e^{-2s}}{s^2} + \frac{4e^{-2s}}{s}$

$= \frac{1}{s^2} + \frac{3e^{-2s}}{s^2} + \frac{6e^{-2s}}{s} + \frac{2e^{-2s}}{s^3}$

$$F(s) = \frac{2e^{-2s}}{s^3} + \frac{3e^{-2s}}{s^2} + \frac{1}{s^2} + \frac{6e^{-2s}}{s}$$



30) $f(t)=\begin{cases}t^2, & 0\le t<1\\ 0, & t\ge 1\end{cases}$

$R/\ f(t)=t^2\left(1-u(t-1)\right);\quad F(s)=\dfrac{2-e^{-s}\left(2+2s+s^2\right)}{s^3}$

$f(t) = t^2[u(t-0)-u(t-1)]$

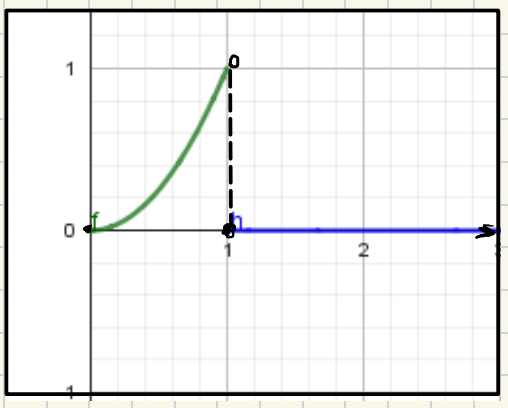
$= t^2u(t) - t^2u(t-1)$

$= t^2 - (t+1)^2u(t-1)$ $\qquad a=1$

$= \mathcal{L}\{t^2\} - e^{-s}\mathcal{L}(t^2+2t+1)$

$= \frac{2}{s^3} - e^{-s}\left(\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}\right)$

$$F(s) = \frac{2-e^{-s}(2+2s+s^2)}{s^3}$$



Made with Goodnotes

En los problemas 34 - 48, determinar L^1{F(s)}

34) $F(s)=\dfrac{\left(1+e^{-2s}\right)^2}{s+2}$

$= \dfrac{1+2e^{-2s}+e^{-4s}}{s+2} = \dfrac{1}{s+2} + \dfrac{2e^{-2s}}{s+2} + \dfrac{e^{-4s}}{s+2} = \dfrac{1}{s+2} + e^{-2s}\left(\dfrac{2}{s+2}\right) + e^{-4s}\left(\dfrac{1}{s+2}\right)$

$=$ sea $s = s+2$ $\quad F(s) = \dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s}\right) + e^{-4s}\left(\dfrac{1}{s}\right)$

$\therefore e^{-2t}$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + e^{-2s}\mathcal{L}^{-1}\left(\frac{2}{s}\right) + e^{-4s}\mathcal{L}^{-1}\left(\frac{1}{s}\right)$$

$$= e^{-2t} + 2e^{-2(t-2)}u(t-2) + e^{-2(t-4)}u(t-4)$$

35) $F(s)=\dfrac{(s)e^{\frac{-4\pi s}{5}}}{s^2+25}$

$R/\ f(t)=u\left(t-\dfrac{4}{5}\pi\right)\cos(5t)$

$= e^{-\left(\frac{4\pi}{5}\right)s} \cdot \mathcal{L}^{-1}\left[\dfrac{s}{s^2+25}\right] = u\left(t-\frac{4\pi}{5}\right)\cos(5t) = u\left(t-\frac{4\pi}{5}\right)\cos(5t+4\pi)$

$a = \frac{4\pi}{5}$ $\quad u(t-a)f(t)$

$t = t - \frac{4\pi}{5} + \frac{4\pi}{5}$

$5t = 5\left(t-\frac{4\pi}{5}\right) + 4\pi$

$5t = 5t + 4\pi$

$\cos(5t)\cos(4\pi) - \operatorname{sen}(5t)\operatorname{sen}(4\pi)$ (1, 0)

$= \cos(5t)$

$\therefore \cos(5t)\,u\left(t-\frac{4\pi}{5}\right)$

$f(t) = \cos(5t)\,u\left(t-\frac{4\pi}{5}\right)$

36) $F(s)=\dfrac{e^{-5s}}{s^2(s-1)}$

$R/\ f(t)=\left(4-t+e^{t-5}\right)u(t-5)$

$= e^{-5s}\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\}$

$a=5$ $\quad u(t-a)f(t-a)$

RESOLVIENDO F(s) POR FRACCIONES PARCIALES

$$\frac{1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1}$$

Si $s=1$

$C = \left.\dfrac{1}{s^2}\right|_{s=1}$ $\quad C=1$

$$\frac{1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1}$$

$1 = As(s-1) + B(s-1) + Cs^2$

$1 = As^2 - As + Bs - B + Cs^2$

$A + C = 0$ $\quad -A + B = 0$ $\quad 1 = -B$

$-1 + 1 = 0$ // $\quad A = B$ $\quad B = -1$

$A = -1$

$$= -\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}$$

$$\mathcal{L}^{-1}\left\{-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}\right\} = -1 - t + e^t$$

$f(t) = e^t - t - 1$ $\quad u = t-5$

$= e^{t-5} - (t+5) - 1$

$= (e^{t-5} - t + 1)$

$\therefore f(t) = (e^{t-5} - t + 1)u(t-5)$

37) $F(s)=\dfrac{e^{-s}\left(1-e^{-s}\right)}{s^2\left(s^2+1\right)}$

$= \dfrac{e^{-s} - e^{-2s}}{s^2(s^2+1)} = \dfrac{e^{-s}}{s^2(s^2+1)} - \dfrac{e^{-2s}}{s^2(s^2+1)}$

$e^{-1s}\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s^2+1)}\right\}$

$a=1$ $\quad f(t-a)u(t-a)$

$$\frac{1}{s^2(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1}$$

$1 = As(s^2+1) + B(s^2+1) + (Cs+D)s^2$

$1 = As^3 + As + Bs^2 + B + Cs^3 + Ds^2$

$A + C = 0$ //

$B + D = 0 \rightarrow D = -1$

$A = 0$ //

$B = 1$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s^2+1}\right\} = (t - \operatorname{sen}(t))$$

$$u(t-2)\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\} = [t + \operatorname{sen}(t)]$$

LLEVANDOLO A FORMA ESCALON UNITARIO:

$u(t-1)[t-\operatorname{sen}(t)] - u(t-2)[t-\operatorname{sen}(t)]$

$$f(t) = (t-1)u(t-1) - \operatorname{sen}(t-1)u(t-1) - (t-2)u(t-2) - \operatorname{sen}(t-2)u(t-2)$$

Made with Goodnotes

42) $F(s)=\dfrac{\left(e^{-s}+1\right)^2}{s^2}$

$R/f(t)=(t-2)u(t-2)+2(t-1)u(t-1)+t$

$= \dfrac{e^{-2s}+2e^{-s}+1}{s^2} = e^{-2s}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+2e^{-s}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$

$t\,u(t-2)+2t\,u(t-1)+t$

$(t-2)u(t-2)+2(t-1)u(t-1)+t$

43) $F(s)=\dfrac{e^{-5s}}{(s-2)^4}$

$R/f(t)=\dfrac{e^{2(t-5)}}{6}(t-5)^3u(t-5)$

$= e^{-5s}\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^4}\right\} = e^{-5s}\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^4}\right\}$

$s=s-2 \therefore e^{2t}$

$= e^{-5s}\left[e^{2t}\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)\right] = \frac{e^{2t}}{3!}\left(\frac{3!}{s^4}\right) = \left(\frac{e^{2t}}{6}t^3\right)e^{-5s} = f(t)$

PASANDOLO A LA FORMA ESCALON UNITARIO

$\left(\dfrac{e^{2t}}{6}t^3\right)e^{-5s} \quad u(t-5)$

$f(t)=\dfrac{e^{2(t-5)}}{6}\cdot(t-5)^3\,u(t-5)$

45) $F(s)=\dfrac{3e^{-2s}}{(s+3)(s+1)}$

$= e^{-2s}\mathcal{L}^{-1}\left\{\frac{3}{(s+3)(s+1)}\right\}$

$A = \left.\frac{3}{s+1}\right|_{s=-3} \quad A=-\frac{3}{2}$

$= \frac{3}{(s+3)(s+1)} = \frac{A}{s+3}+\frac{B}{s+1} \quad B = \left.\frac{3}{s+3}\right|_{s=-1} \quad B=\frac{3}{2}$

$= \mathcal{L}\left\{-\frac{3}{2}\left(\frac{1}{s+3}\right)+\frac{3}{2}\left(\frac{1}{s+1}\right)\right\}$

$= \left(-\frac{3}{2}e^{-3t}+\frac{3}{2}e^{-t}\right)e^{-2s}$

PASANDOLO A FORMA DE ESCALON UNITARIO

$f(t)=\dfrac{3}{2}\left(-e^{-3(t-2)}+e^{-(t-2)}\right)u(t-2)$

46) $F(s)=\dfrac{6e^{-4s}\left(e^{-s}+1\right)^3}{(s-1)^4}$

1 / 1 2 1 / 1 3 3 1

$(e^{-3s}+3e^{-2s}+3e^{-s}+1)(6e^{-4s})$

$s=s-1 \therefore e^{t}$

$= \dfrac{6e^{-7s}+18e^{-6s}+18e^{-5s}+6e^{-4s}}{(s-1)^4} = 6e^{-7s}\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)+18e^{-6s}\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)+18e^{-5s}\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)+6e^{-4s}\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)$

$f(t) = e^{-7s}e^{t}t^3+3e^{-6s}e^{t}t^3+3e^{-5s}e^{t}t^3+e^{-4s}e^{t}t^3$

PASANDOLO A LA FORMA DE ESCALON UNITARIO

$f(t)=e^{t-7}(t-7)^3u(t-7)+3e^{(t-6)}(t-6)^3u(t-6)+3e^{(t-5)}(t-5)^3u(t-5)+e^{(t-4)}(t-4)^3u(t-4)$

47) $F(s)=\dfrac{(s+1)e^{-s}}{s^2\left(s^2+1\right)}$

$= e^{-s}\mathcal{L}^{-1}\left\{\frac{s+1}{s^2(s^2+1)}\right\} \quad \frac{s+1}{s^2(s^2+1)} = \frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+1}$

$s+1 = As(s^2+1)+B(s^2+1)+(Cs+D)s^2$

$s+1 = As^3+As+Bs^2+B+Cs^3+Ds^2$

$A+C=0 \quad A=1 \quad B+D=0 \quad B=1 \quad D=-1 \quad C=-1$

$R/f(t)=(t-\cos(t-1)-sen(t-1))u(t-1)$

$\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+1} = \left\{\frac{1}{s}+\frac{1}{s^2}-\frac{s}{s^2+1}-\frac{1}{s^2+1}\right\}$

APLICANDO TRANSFORMADA INVERSA DE LAPLACE:

$f(t)=(1-t-\cos(t)-sen(t))e^{-s} \quad a=1 \quad u(t-1)$

PASANDOLA A FORMA DE ESCALON UNITARIO

$f(t)=[1+t-1-\cos(t-1)-sen(t-1)]u(t-1)$

$f(t)=[t-\cos(t-1)-sen(t-1)]u(t-1)$

Made with Goodnotes

II. Derivada de la transformada ——— **En los ejercicios del 49 al 63, determinar F (s) .**

TEOREMA 4.8 — Derivadas de transformadas

Si $F(s) = \mathcal{L}\{f(t)\}$ y $n = 1, 2, 3, \ldots$, entonces

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s).$$

$\mathcal{L}\{t^1 \operatorname{sen}(2t)\} = (-1)^1 \frac{d}{ds}\left(\mathcal{L}\{\operatorname{sen}(2t)\}\right) = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

$= -\left(\frac{0(s^2+4) - 2(2s)}{(s^2+4)^2}\right) = \frac{4s}{(s^2+4)^2}$

49) $f(t) = te^{-5t}\cos(4t)$ $= -(1)\frac{d}{ds}\mathcal{L}\{e^{-5t}\cos(4t)\}$ $\quad s = s+5$

Aplicando el primer T.T

$-\frac{d}{ds}\left(\mathcal{L}(\cos 4t)\Big|_{s=s+5}\right) = \frac{s}{s^2+16}\Big|_{s=s+5} = \frac{s+5}{(s+5)^2+16} = \frac{d}{ds}\left(\frac{s+5}{s^2+10s+41}\right)$

$= -\left(\frac{(1)(s^2+10s+41) - (s+5)(2s+10)}{((s+5)^2+16)^2}\right)$

$= -s^2 - 10s - 41 + 2s^2 + 20s + 50 = \boxed{\frac{s^2+10s+9}{((s+5)^2+16)^2}}$

$= -\left(\frac{(s^2+10s+41) - 2s^2 - 20s - 50}{((s+5)^2+16)^2}\right)$

50) $f(t) = t\operatorname{sen}(8t)$

$= -\frac{d}{ds}\left(\frac{8}{s^2+64}\right) = -\left(\frac{-8(2s)}{(s^2+64)^2}\right) = \boxed{\frac{16s}{(s^2+64)^2}}$

51) $f(t) = t^2 \operatorname{sen}(3t)$

$= (-1)^2 \frac{d^2}{ds^2}\left(\mathcal{L}\{\operatorname{sen}(3t)\}\right) = \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{3}{s^2+9}\right)\right) = \frac{d}{ds}\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{-6(s^2+9)^2 + 6s(2(s^2+9)(2s))}{[(s^2+9)^2]^2}$

$= \frac{-6(s^2+9)^2 + 24s^2(s^2+9)}{(s^2+9)^4} = \frac{-6(s^2+9)((s^2+9) - 4s^2)}{(s^2+9)^4}$

$= -6(s^2+9) + 24s^2 = -6s^2 - 54 + 24s^2 = 18s^2 - 54 \rightarrow \frac{18(s^2-3)}{(s^2+9)^3}$

54 | 18
24 | 3
0

$= \boxed{\frac{18(s^2-3)}{(s^2+9)^3}}$

53) $f(t) = te^{2t}\cos(3t)$ *aplicando el primer TT* $\quad s = s-2$

$= -\frac{d}{ds}\left[\mathcal{L}\{e^{2t}\cos(3t)\}\right] \quad \mathcal{L}\{\cos(3t)\}\Big|_{s=s-2}$

$= \frac{s-2}{(s-2)^2+9} = \frac{s-2}{s^2-4s+13}$

$= -\left[\frac{s-2}{s^2-4s+13}\right] = -\left[\frac{(1)(s^2-4s+13) - (s-2)(2s-4)}{(s-4s+13)^2}\right]$

$= -(s^2 - 4s + 13 - 2s^2 + 8s - 8)$

$= -(-s^2 + 4s + 5) = \boxed{\frac{s^2-4s-5}{(s-4s+13)^2}}$

57) $f(t) = (t-2)^2 \operatorname{sen}(t)$

$= (t^2 - 4t + 4)\operatorname{sen}(t) = t^2\operatorname{sen}(t) - 4t\operatorname{sen}(t) + 4\operatorname{sen}(t)$

$= (-1)^2 \frac{d}{ds}\left(\frac{d}{ds}\left(\mathcal{L}\{\operatorname{sen}(t)\}\right)\right) - 4(-1)\frac{d}{ds}\left(\mathcal{L}\{\operatorname{sen}(t)\}\right) + \frac{4}{(s^2+1)}$

$= \frac{d}{ds}\left(\frac{d}{ds}\left(\mathcal{L}\{\operatorname{sen}(t)\}\right)\right) + 4\frac{d}{ds}\left(\mathcal{L}\{\operatorname{sen}(t)\}\right)$

$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right] + 4\frac{d}{ds}\left[\frac{1}{s^2+1}\right] \rightarrow = \frac{d}{ds}\left[\frac{-2s}{(s^2+1)^2}\right] + 4\left[\frac{-2s}{(s^2+1)^2}\right]$

$= \frac{-2(s^2+1)^2 + 2s(2(s^2+1)(2s))}{(s^2+1)^4}$

$= \frac{-2(s^2+1)^2 + 8s^2(s^2+1)}{(s^2+1)^4}$

$= \frac{-2(s^2+1)[(s^2+1) + 8s^2]}{(s^2+1)^4}$

$\frac{-2s^2-2+8s^2}{(s^2+1)^3} = \frac{6s^2-2}{(s^2+1)^3} - \frac{8s}{(s^2+1)^2} = \frac{6s^2-2-8s(s^2+1)}{(s^2+1)^3}$

$= \frac{-8s^3+6s^2-8s-2}{(s^2+1)^3} + \frac{4}{(s^2+1)} = \frac{-8s^3+6s^2-8s-2+4(s^2+1)^2}{(s^2+1)^3}$

$\frac{-8s^3+6s^2-8s-2+4s^4+8s^2+4}{(s^2+1)^3} = \frac{4s^4-8s^3+14s^2-8s+2}{(s^2+1)^3}$

$$F(s) = \frac{2(2s^4 - 4s^3 + 7s^2 - 4s + 1)}{(s^2+1)^3}$$

Made with Goodnotes

58) $f(t)=\dfrac{sen(t)}{t}$

$= \quad t f(t) = sen(t)$

$\mathcal{L}\{t^1 f(t)\} = \mathcal{L}\{sen(t)\}$

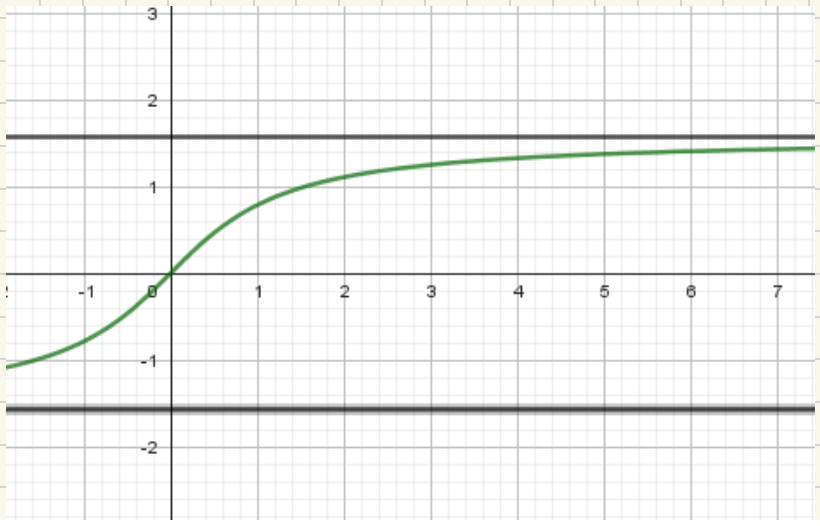
$-\dfrac{d}{ds}\left(\mathcal{L}\{f(t)\}\right) = \dfrac{1}{s^2+1}$

$\dfrac{d}{ds}\left(\mathcal{L}\{f(t)\}\right) = -\dfrac{1}{s^2+1^2}$

aplicando variables separables

$\int d[F(s)] = -\int \dfrac{1}{s^2+1^2}\, ds$

$F(s) = -\tan^{-1}(s) + C$



aplicando limites

$\lim_{s\to\infty} F(s) = -\dfrac{1}{\tan(\infty)} + C = 0$

$= -\dfrac{\pi}{2} + C = 0$

$C = \dfrac{\pi}{2}$

$F(s) = \dfrac{\pi}{2} - \tan^{-1}(s)$

59) $f(t)=\dfrac{d}{dt}\int_0^{t^2}\cos\left(3\sqrt{x}\right)dx, \; t>0 \qquad R/F(s)=\dfrac{2\left(s^2-9\right)}{\left(s^2+9\right)^2}$

aplicando el teorema fundamental del calculo:

$\dfrac{d}{dt}\int_0^{t^2}\cos(3\sqrt{x})\,dx = \cos(3t)\dfrac{d}{dt}(t^2)$

$= 2\cos(3t)t$

$t>0 \qquad F(t) = 2t\cos(3t)$

$= -2\dfrac{d}{ds}\left(\mathcal{L}\{\cos(3t)\}\right)$

$= -2\dfrac{d}{ds}\left(\dfrac{s}{s^2+9}\right) = -2\left[\dfrac{s^2+9-s(2s)}{(s^2+9)^2}\right] = -2\left[\dfrac{-s^2+9}{(s^2+9)^2}\right]$

$= -2\left[\dfrac{s^2+9-2s^2}{(s^2+9)^2}\right] = \dfrac{2s^2-18}{(s^2+9)^2}$

$= \dfrac{2(s^2-9)}{(s^2+9)^2}$

60) $f(t)=(t-\pi)\,sen(t-\pi)\,u(t-\pi)$

$u(t-\pi) \rightsquigarrow e^{-\pi s}$

$f(t) = e^{-\pi s}\mathcal{L}\{t\,sen(t)\}$

$F(s) = e^{-\pi s}\left[-\dfrac{d}{ds}\mathcal{L}\{sen(t)\}\right] = \left(\dfrac{1}{s^2+1}\right)$

$= -e^{-\pi s}\left[\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right)\right] = \dfrac{2e^{-\pi s}s}{(s^2+1)^2}$

$F(s) = \dfrac{2e^{-\pi s}}{(s^2+1)^2}$

61) $\mathcal{L}\{t\,sen(3t)\}$

$= -\dfrac{d}{ds}\mathcal{L}\{sen(3t)\} = -\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right) = -\left[\dfrac{d/ds(3)(s^2+9) - 3\,d/ds(s^2+9)}{(s^2+9)^2}\right]$ (d/ds(3) → 0; d/ds(s²+9) → 2s)

$F(s) = \dfrac{6s}{(s^2+9)^2}$

62) $\mathcal{L}\{t^2\cosh(t)\} \qquad R/F(s)=\dfrac{2s\left(s^2+3\right)}{\left(s^2-1\right)^3}$

$= \dfrac{d}{ds}\left[\dfrac{d}{ds}\left\{\mathcal{L}\{\cosh(t)\}\right]\right] = \dfrac{d}{ds}\left[-\dfrac{(s^2+1)}{(s^2-1)^2}\right]$; $\dfrac{d}{ds}\left[-\dfrac{(s^2+1)}{(s^2-1)^2}\right]$

$-\dfrac{d}{ds}\left[\dfrac{(s^2+1)}{(s^2+1)^2}\right] = (2s)(s^2-1)^2 - (s^2+1)(2(s^2-1)(2s)) = 2s(s^2-1)^2 - 4s(s^2+1)(s^2-1) = \dfrac{2s(s^2-1)[(s^2-1)-2s(s^2+1)]}{(s^2-1)^3}$

$\dfrac{2s(s^2-1)[(s^2-1)-2s(s^2+1)]}{(s^2-1)^3} = 2s(s^2-1) - 4s(s^2+1) = 2s^3-2s-4s^3-4s = (-2s^3-6s)(-1) = \dfrac{2s(s^2+3)}{(s^2-1)^3}$

$F(s) = \dfrac{2s(s^2+3)}{(s^2-1)^3}$

Made with Goodnotes

63) $\mathcal{L}\{te^{2t}\cos(6t)\}$ $= -\frac{d}{ds}\left[\mathcal{L}\{e^{2t}\cos(6t)\}\right]$ aplicando el 1 T.T $\mathcal{L}\{\cos(6t)\}\big|_{s=s-2}$

$= -\frac{d}{ds}\left[\frac{s-2}{s^2-4s+40}\right]$ $= \frac{s-2}{s^2-4s+40}$ $s^2-4s+4+36$

$(s-8)(s+4)$

$s^2+4s-8s-32$
$s^2-4s-32$

$= -\left[\frac{(1)(s^2-4s+40)-(s-2)(2s-4)}{(s^2-4s+40)^2}\right] = -\left[\frac{s^2-4s+40-2s^2+8s-8}{(s-4s+40)^2}\right]$

$$F(s) = \frac{s^2-4s-32}{(s-4s+40)^2}$$

Made with Goodnotes